JOURNAL
OF THE
WASHINGTON ACADEMY OF SCIENCES
VOL. VI DECEMBER 4, 1916 No. 20

MATHEMATICS.—*Note on relativity: The geometric potential.* EDWIN BIDWELL WILSON, Massachusetts Institute of Technology. (Communicated by Arthur L. Day.)

1. In our treatment[1] of the principle of relativity Lewis and I introduced as fundamental the extended (i.e., four-dimensional) vector **m** of which the space and time components, when once a time-axis has been arbitrarily selected, are the ordinary retarded vector potential **a** and the retarded scalar potential φ. We called the vector **m** the extended vector potential, and by its differentiation we obtained the electromagnetic field equations. This is the converse of the usual procedure, which is to regard the field equations as fundamental and to introduce the retarded potentials as "certain auxiliary functions on which the electric and magnetic forces may be made to depend."[2]

We built up the potential for a distributed charge from that for a point charge and reduced the potential of a point charge to the product of the charge and a vector **p**, which may be called a geometric potential because of its definition solely by geometric means. To find the potential **p** at a point Q (of the four-dimensional manifold) and due to a curve δ which is the space-time locus of a moving charge, the first step is to draw the backward singular cone with vertex Q and determine its intersection O with the curve δ; then draw at O the forward unit tangent **w** to the curve, and let the perpendicular from Q

[1] WILSON, EDWIN B., and LEWIS, GILBERT N., *The space-time manifold of relativity; the non-euclidean geometry of mechanics and electromagnetics*, Proc. Amer. Acad. Arts Sci., **48**: 389–507. 1912.

[2] See, for example, LORENTZ, The Theory of Electrons, p. 19. What Lorentz here calls electric and magnetic forces are what we call field intensities.

to this tangent be R. If then $\mathbf{l}$ denotes the vector OQ, the singular vector from the acting point to the point at which the potential is desired, the potential[3] $\mathbf{p}$ is

$$\mathbf{p} = \frac{\mathbf{l}}{R} = -\frac{\mathbf{l}}{\mathbf{l}\cdot\mathbf{w}} \tag{1}$$

The vector $\mathbf{p}$ satisfies the two fundamental conditions

$$\Diamond \cdot \Diamond \mathbf{p} = 0, \quad \Diamond \cdot \mathbf{p} = 0, \tag{2}$$

of which the first is the wave equation needed to represent the fact of the propagation of an electromagnetic disturbance, and the second is the condition usually imposed upon the auxiliary functions $\mathbf{a}$ and φ to make their determination complete.[4]

To validate the selection of (1) as definition of the potential, a definition which thus far apparently has nothing but simplicity in its favor, we may cite our proof that (1) is, apart from a numerical multiplier, the only possible form for the potential $\mathbf{p}$ which satisfies (2) and depends only on $\mathbf{l}$ and $\mathbf{w}$, that is, on the retarded position and velocity but not on the acceleration.

2. I wish now to remove from the hypotheses the condition $\Diamond \cdot \mathbf{p} = 0$ and to state the theorem:

The only possible choice for the geometrical potential $\mathbf{p}$, dependent only on $\mathbf{l}$ and $\mathbf{w}$ but not on the derivatives of $\mathbf{w}$, and subject to satisfying the wave equation $\Diamond \cdot \Diamond \mathbf{p} = 0$, is $\mathbf{p} = A\mathbf{l}/R$, where A is a constant.[5]

When we have proved this theorem we have, from the point of view of relativity, a completely rational basis for the theory of the potential and field of the point charge,[6] and through it,

[3] This is the definition given by MINKOWSKI in his *Raum und Zeit*, Gesammelte Werke, vol. 2, p. 442.

[4] See, for example, LORENTZ, loc. cit., p. 239.

[5] PAGE, L., in *Relativity and ether*, Amer. J. Sci., **37**: 169–187, 1914, apparently reaches a similar conclusion in a totally different way; but it is difficult to compare the arguments.

[6] Even if we believe that electricity always occurs in continuous distributions, that is, that electrons are continuous surface or volume spreads of electricity with appropriate densities, it is convenient to have a rational theory of the point charge for those investigations in which the size of the electron is negligible, and particularly as the density within or upon the electron is unknown.

by integration, for the theory of continuous distributions of electricity.

Before we can give the proof, we must make precise the meaning of the statement that $\mathbf{p}$ is dependent only on $\mathbf{l}$ and $\mathbf{w}$. Geometrically[7] speaking, $\mathbf{l}$ and $\mathbf{w}$ determine a plane, and thus also a second plane completely perpendicular to their plane, but they determine no particular vector in this second plane or in their own plane.[8] Hence, if a vector $\mathbf{p}$ is to depend on $\mathbf{l}$ and $\mathbf{w}$ alone, it must lie in their plane. The scalar products of $\mathbf{l}$ and $\mathbf{w}$ by themselves are

$$\mathbf{l}\cdot\mathbf{l} = 0, \quad \mathbf{l}\cdot\mathbf{w} = -R, \quad \mathbf{w}\cdot\mathbf{w} = -1.$$

Hence the function $\mathbf{p}$ must take the form

$$\mathbf{p} = \varphi(R)\,\mathbf{w} + f(R)\,\mathbf{l}. \tag{3}$$

To show that $\mathbf{p}$ reduces to the form $A\,\mathbf{l}/R$, we have merely to substitute the general form (3) in the equation $\diamondsuit\cdot\diamondsuit\,\mathbf{p} = 0$ and see that the only possibilities are $\varphi(R) = A/R$, $f(R) = 0$. Now if f is a scalar and $\mathbf{u}$, $\mathbf{v}$ are two vectors,

$$\begin{aligned}
\diamondsuit\cdot\diamondsuit(f\mathbf{v}) &= (\diamondsuit\cdot\diamondsuit f)\mathbf{v} + 2\,\diamondsuit f\cdot\diamondsuit\mathbf{v} + f\,\diamondsuit\cdot\diamondsuit\mathbf{v} \\
\diamondsuit\cdot\diamondsuit(\mathbf{u}\cdot\mathbf{v}) &= \mathbf{v}\cdot(\diamondsuit\cdot\diamondsuit\mathbf{u}) + 2\,\diamondsuit\mathbf{u}:\diamondsuit\mathbf{v} + \mathbf{u}\cdot(\diamondsuit\cdot\diamondsuit\mathbf{v}) \\
\diamondsuit\cdot\diamondsuit f(R) &= f''(R)\,\diamondsuit R\cdot\diamondsuit R + f'(R)\,\diamondsuit\cdot\diamondsuit R
\end{aligned}$$

With the formulas that we have established (§44, loc. cit.), namely

$$\diamondsuit\,\mathbf{w} = \frac{1}{R}\mathbf{lc} \qquad \diamondsuit\,\mathbf{c} = -\frac{1}{R}\frac{d\mathbf{c}}{ds}$$

$$\diamondsuit\,\mathbf{l} = I + \frac{1}{R}\mathbf{lw} \qquad \diamondsuit R = -\mathbf{w} + \frac{1+\mathbf{l}\cdot\mathbf{c}}{R}\mathbf{l}$$

where $\mathbf{c}$ is the retarded curvature $d\mathbf{w}/ds$ of the space-time locus

[7] We might discuss this question more in detail as H. BURKHARDT does the corresponding general problem for three dimensional vector analysis in *Ueber Functionen von Vectorgrössen, welche selbst wieder Vectorgrössen sind. Eine Anwendung invariantentheoretischer Methoden auf eine Frage der mathematischen Physik.* Math. Ann., **43**: 197–215. 1893. For our present purposes this seems hardly necessary.

[8] The plane of $\mathbf{l}$ and $\mathbf{w}$ and the plane completely perpendicular to it cut the singular cone in pairs of lines which are respectively real and imaginary, but no vectors along these directions are determined.

of the charge, and I is the idemfactor, it is easy to show that

$$\Diamond\cdot\Diamond\mathbf{p} = -2\mathbf{c}\left(\varphi' + \frac{\varphi}{R}\right) + \mathbf{w}\left[\frac{2f}{R} + \left(\varphi'' + \frac{2\varphi'}{R}\right)(1 + 2\,\mathbf{l}\cdot\mathbf{c})\right]$$
$$+\mathbf{l}\left[\left(f'' + \frac{2f'}{R}\right)(1 + 2\,\mathbf{l}\cdot\mathbf{c}) + 2f'(1 + \mathbf{l}\cdot\mathbf{c})\right]$$

If this is to vanish, $\varphi' + \varphi/R = 0$, that is

$$\varphi = \frac{A}{R} \qquad f = 0$$

3. As the potential has turned out homogeneous of degree zero in $\mathbf{w}$, we may write

$$\mathbf{p} = -\frac{\mathbf{w}}{\mathbf{l}\cdot\mathbf{w}} = -\frac{\mathbf{u}}{\mathbf{l}\cdot\mathbf{u}}$$

where $\mathbf{u}$ is any tangent to the curve δ. If we define dq by the relation $d\mathbf{r} = \mathbf{u}\,dq$, where $d\mathbf{r}$ is the increment along the curve, we have determined a parametric representation of the curve so that $\mathbf{u} = d\mathbf{r}/dq$, which is analogous to $\mathbf{w} = d\mathbf{r}/ds$, but more general in that it would be applicable to curves of zero length.

The equations for the derivatives would now become

$$\Diamond\mathbf{u} = \frac{\mathbf{l}\mathbf{c}^1}{\mathbf{l}\cdot\mathbf{u}} \qquad \Diamond\mathbf{l} = I - \frac{\mathbf{l}\mathbf{u}}{\mathbf{l}\cdot\mathbf{u}}$$

$$\Diamond(\mathbf{l}\cdot\mathbf{u}) = (\Diamond\mathbf{l})\cdot\mathbf{u} + (\Diamond\mathbf{u})\cdot\mathbf{l} = \mathbf{u} - \frac{\mathbf{u}\cdot\mathbf{u}}{\mathbf{l}\cdot\mathbf{u}}\mathbf{l} + \frac{\mathbf{l}\cdot\mathbf{c}^1}{\mathbf{l}\cdot\mathbf{u}}\mathbf{l}$$

where $$\mathbf{c}^1 = d\mathbf{u}/dq$$

Then

$$\Diamond\mathbf{p} = \frac{\mathbf{u}\,\mathbf{u}}{(\mathbf{l}\cdot\mathbf{u})^2} - \frac{\mathbf{u}\cdot\mathbf{u}}{(\mathbf{l}\cdot\mathbf{u})^3}\mathbf{l}\,\mathbf{u} + \frac{\mathbf{l}\cdot\mathbf{c}^1}{(\mathbf{l}\cdot\mathbf{u})^3}\mathbf{l}\,\mathbf{u} - \frac{\mathbf{l}\,\mathbf{c}^1}{(\mathbf{l}\cdot\mathbf{u})^2}$$

and

$$\Diamond\times\mathbf{p} = -\frac{\mathbf{u}\cdot\mathbf{u}}{(\mathbf{l}\cdot\mathbf{u})^3}\mathbf{l}\times\mathbf{u} + \frac{\mathbf{l}\cdot\mathbf{c}^1}{(\mathbf{l}\cdot\mathbf{u})^3}\mathbf{l}\times\mathbf{u} - \frac{\mathbf{l}\times\mathbf{c}^1}{(\mathbf{l}\cdot\mathbf{u})^2}$$

$$\mathbf{P} = \Diamond\times\mathbf{p} = -\frac{\mathbf{u}\cdot\mathbf{u}}{(\mathbf{l}\cdot\mathbf{u})^3}\mathbf{l}\times\mathbf{u} + \frac{\mathbf{l}\times[\mathbf{l}\cdot(\mathbf{u}\times\mathbf{c}^1)]}{(\mathbf{l}\cdot\mathbf{u})^3}$$

The vector $\mathbf{P}$, of the second sort, is the (geometric) field[9] set

[9] WILSON and LEWIS, loc. cit., p. 460.

up by the potential; the first term varies inversely as the square of the interval of **l**, and the second term inversely as the interval itself. When $\mathbf{u} = \mathbf{w}$ the result reduces to that previously found.[10] The interest attaching to the present form is that it is applicable to the case in which the curve is a singular curve, that is, to the path of a particle of light, whereas the previous form was applicable only to curves that could be described by electrons or material particles.

If we consider the vector **u** as a singular vector, the first term in **P** drops out by virtue of $\mathbf{u}.\mathbf{u} = 0$. We have therefore the following result:

In the (geometric) field of a particle of light that portion which corresponds to Coulomb's law for the field of an ordinary particle vanishes identically.

The field **P** vanishes entirely unless the particle of light (assumed to be moving with the normal velocity of light in free space) travels in a curved path so that $\mathbf{c}^1$ is not parallel to **u**.

MINERALOGY.—*Lorettoite, a new mineral.*[1] ROGER C. WELLS and ESPER S. LARSEN, Geological Survey.

A specimen, furnished by Mr. Frank L. Hess of the United States Geological Survey and received by him from Mr. I. N. Wilconson of Loretto, Tennessee, proved on analysis to differ chemically from any known mineral. A specimen labeled "massicot," without a location, in the collections of the University of California, differs in its optical properties from any known mineral, and a later optical examination of the Loretto mineral showed the essential identity of the two. The name lorettoite is proposed for the mineral, from its occurrence near Loretto, Tennessee.

Physical properties. The mineral is in flat, compact pieces, up to an inch thick, and apparently occurs in thin seams. It has a bladed structure and a very perfect cleavage along these blades. The blades commonly extend across the specimen and

[10] WILSON and LEWIS, loc. cit., p. 464.

[1] Published with the permission of the Director of the U. S. Geological Survey.

are about half a millimeter in width. The specific gravity of the Loretto specimen, as determined by the picnometer method, is 7.39; that of the University of California specimen, as determined with a Joly balance, is 7.65. The cloudiness of the Loretto specimen is due to minute gas cavities which may account for the apparently low specific gravity. The Loretto specimen fuses readily in the flame of a candle (F = 1) to a mass which on cooling is a yellow, crystalline bead. It has a hardness of about 3. Its luster is adamantine, its color honey-yellow, and its streak pure yellow.[2] It is optically negative, sensibly uniaxial, and the optic axis is normal to the cleavage. The indices of refraction as measured in sulphur-selenium melts are, for the Loretto specimen:

$$\omega_{Li} = 2.40 \pm 0.02$$
$$\epsilon_{Li} = 2.37 \pm 0.02$$

and for the specimen at the University of California:

$$\omega_{Li} = 2.35 \pm 0.02$$
$$\epsilon_{Li} = 2.33 \pm 0.02$$

The specimen from the University of California is less clouded than that from Loretto, and basal sections show a delicate cross-grating at 90° due to the minute gas inclusions collected along certain planes. The mineral is therefore probably tetragonal with very perfect basal cleavage.

Chemical properties. Lorettoite dissolves easily in hot dilute nitric acid, leaving only a very slight residue. It dissolves slowly in hot dilute hydrochloric acid, and lead chloride separates from the solution on cooling. Sulfuric acid decomposes it very slowly. It is not appreciably soluble in hot water. The slight effervescence of the Loretto specimen with acids is due to a small amount of carbonate.

A microscopic examination of the powder analyzed showed a little impurity, estimated at 2 per cent, consisting chiefly of carbonate and a little of an opaque mineral, lead gray in reflected

[2] Following Ridgway's Color Standards and Nomenclature, 1912, the color is honey-yellow (19″—), and the streak strontium-yellow (23′—).

light, probably galena or metallic lead. The results of a chemical analysis of lorettoite are given in Table 1. The material on which the second chlorine determination was made contained very little impurity.

TABLE 1

COMPOSITION OF LORETTOITE FROM LORETTO, TENNESSEE

(R. C. Wells, analyst)

	1	2
Insoluble	0.58	
PbO	93.98	
Cl	3.98	4.09
P_2O_5	0.11	
CaO	0.48	
MgO	0.56	
Al_2O_3	0.08	
ZnO	0.31	
CO_2	0.20	
Br, I, F	None	
H_2O	0.03	
	100.31	
Less O eq. of Cl	0.90	
	99.41	

Chlorine was determined by gently fusing the mineral with sodium carbonate and, after carefully neutralizing the cold aqueous extract of the melt with nitric acid, precipitating and weighing the chlorine as silver chloride. A determination of chlorine in selected material, almost entirely homogeneous, from the California specimen gave 4.94 per cent.

If all the constituents except chlorine, lead, and oxygen in the complete analysis are regarded as extraneous, the composi-

TABLE 2

	(a)	(b)	(c)
PbO	83.72	80.62	82.80
$PbCl_2$	16.28	19.38	17.20
	100.00	100.00	100.00

tion reduces to (*a*), Table 2. If the California specimen is assumed to contain only lead, oxygen, and chlorine, the composition would be as shown under (*b*). Under (*c*) is given the theoretical percentage for the formula $6PbO.PbCl_2$.

It would, perhaps, be rash to claim that the evidence at hand establishes the existence of a definite chemical compound with the formula $6PbO.PbCl_2$, but the composition of the lorettoite approaches more nearly to the requirements of this than of any other simple formula.

Summary. Lorettoite occurs in honey-yellow masses made up of rather coarse fibers or blades. It is probably tetragonal in crystallization and has a very perfect basal cleavage. Its specific gravity is about 7.6, its hardness about 3, and its fusibility about 1. Its luster is adamantine and its streak is pure yellow. It is sensibly uniaxial, optically negative, and its indices of refraction are: $\omega_{Li} = 2.40$, $\epsilon_{Li} = 2.37$. It dissolves readily in acid and has the approximate composition $6PbO.PbCl_2$.

SOIL CHEMISTRY.—*A chemical study of the habitat of the walking fern,* Camptosorus rhizophyllus (*L.*) *Link.*[1] EDGAR T. WHERRY, National Museum.

A problem that often confronts the field geologist is the determination of the calcareous or non-calcareous nature of a given ledge of rock, and while a bottle of acid can be carried along and actual tests of the rock for carbonates performed, the existence of an easily recognizable index-plant, which might be found growing only on calcareous rocks, would be a great advantage. In order to ascertain the possible value in this connection of the plants classed by botanists as calciphilous, the writer decided to make a chemical study of the rocks associated with such a plant. For this purpose the walking fern, *Camptosorus rhizophyllus,* was selected, as it is a fairly common and readily recognized plant and is stated in all of the well known botanical treatises to prefer a calcareous habitat.

Samples of the rocks and soils on which colonies of the plant

[1] Published by permission of the Secretary of the Smithsonian Institution.

were growing were collected at some twenty places in Pennsylvania, Maryland, Virginia, and West Virginia. It is a pleasure to acknowledge the assistance in locating these stations received from Mr. Harold W. Pretz, of Allentown, Pa., Prof. Glenn V. Brown and Prof. N. F. Davis of Bucknell University, Lewisburg, Pa., and Dr. T. C. Stotler and Prof. H. T. McDonald, of Harpers Ferry, W. Va.

The walking fern was found not only on limestone, but also on various rocks not ordinarily classed as calcareous, comprising granite, schist, shale, sandstone, and quartzite, as well as on tree trunks, both living and dead. To make certain that these rocks were actually low in lime, analyses were made on samples collected as near as possible to the roots of the fern plants. Standard methods of analysis were employed, involving decomposition of the rock by evaporation with hydrofluoric acid or by fusion with sodium carbonate, removal of the iron and aluminium by ammonium hydroxide, and precipitation of the calcium as oxalate, followed by ignition and weighing as lime. For completeness, several undoubtedly calcareous rocks were also analyzed, being first dissolved in hydrochloric acid and the lime separated as above outlined. The several rocks were found to vary in lime content from 53.8 per cent down to less than 0.1 per cent; details are given in Table 1, below.

At the outset, then, it was apparent that the walking fern could not be depended on as an index of calcareous rocks. From a theoretical viewpoint, however, plants would be expected to respond not so much to the rock upon which they grow as to the soil which clothes that rock, since it is from the soil that their mineral nutriment is directly obtained. Analysis showing from 30 to 40 per cent of lime in the ash of the plant, it was decided to extend the investigation so as to determine whether the walking fern might perhaps be limited to highly calcareous soils.

That the soil supporting plant growth is not necessarily closely related in composition to the underlying rock has been repeatedly pointed out,[2] yet is not always recognized in studies

[2] Compare, COVILLE, FREDERICK V. *The formation of leafmold.* Journ. Wash. Acad. Sci., **3**: 77. 1913; Ann. Rept. Smithsonian Inst., 1913, 333. 1914.

of plant habitats; attention is therefore again called to it here. The soil may contain: (1) less lime than the underlying rock if (a) it originated on a non-calcareous rock, but has been transported to a more calcareous one, or (b) lime has been leached from it by the rain; (2) more lime than the underlying rock if (a) it originated on a calcareous rock, but has been transported to a less calcareous one, or (b) lime has become concentrated in it by long continued decay of vegetable matter.

To determine which of the above relations holds in the case of the walking fern, samples of the several soils were dried at 105°C., so that they could be more easily handled, and shaken from the network of roots. The soils in almost every case were found to be filled with rock fragments of all sizes, so that some arbitrary classification, on the basis of size, into "rock" and "soil" particles was unavoidable. A sieve with 40 meshes to the centimeter was adopted for this purpose, and the various soils were gently sifted through it, the chips of rock and coarse vegetable matter being thereby removed. The particles passing through this sieve were finely powdered in an agate mortar and used for the analytical work. Two-tenth gram samples of the soils were weighed out into platinum crucibles, and ignited over complete-combustion burners to drive off volatile matter; this varied between 20 and 80 per cent in the several soils studied. The residues were then fused with sodium carbonate, and analyzed for total lime in the usual way; the results are given in Table 1.

The total lime is, however, probably not so important in this connection as the soluble lime, for it is conceivable that even in soils high in total lime only a very minute amount may be present in such a form as to be available to the plants. It was, therefore, decided to test the soils for soluble lime.

It is, of course, impracticable to determine the amount of lime in the natural soil liquid when dealing with such small quantities of material as are available in this case. An excess of water must be added, so that a volume of liquid sufficient for analysis can be obtained. The complexity of soils is so great, and the factors involved so numerous, that it can not be assumed that the composition of a solution obtained by adding

an excess of water to a soil will bear any simple relation to that of the original soil liquid, nor that it will contain any definite fraction of the total soluble lime. Although the analytical results will, therefore, have no absolute numerical significance, their general order of magnitude should show in a rough way the extent to which lime is available to the plant. In order that the results should represent as nearly as possible the natural condition of the soil, the samples to be used for determination of soluble lime were not dried, finely sifted, or pulverized. For the same reason water saturated with carbon dioxide was used for extracting, and the amount of this water was reduced to the smallest possible quantity consistent with convenient analytical operation.

One-gram samples of soil were carefully separated from the roots and placed in centrifuge tubes, 3 cc. of distilled water saturated with carbon dioxide being added; the tubes were shaken gently to insure uniform moistening, stoppered, allowed to stand one hour, and centrifuged for a few minutes, and the lime in the liquid was determined in the usual way. The results obtained are given, along with the others, in Table 1.

Still another feature of the soils, their acidity or alkalinity, seemed worth determining, since there is evidence that some plants are sensitive to relatively slight changes in these factors. The acidity or alkalinity of soils is often expressed in terms of "normality," the equivalent weight of soil being taken as 1000 and water extracts being titrated with standard alkali or acid, using phenolphthalein as indicator; and such normality determinations were made as described below. In addition, to determine the *true* acidity (or alkalinity), the soils were examined by the colorimetric method developed by Dr. L. J. Gillespie of the Bureau of Soils, U. S. Department of Agriculture.[3] The results obtained by this method are presented in two forms, in the last three columns of the table. First the P_H value is given, this being the negative exponent of 10 corresponding to the concentration of hydrogen ions present in the solution. Then, as the P_H figures do not express clearly the relative strengths

[3] Journ. Wash. Acad. Sci., **6**: 7. 1916.

of the acids (or alkalies) to persons not accustomed to thinking in such terms, actual numerical intensities of acid and alkali are also given. These have been obtained by subtracting 7, the P_H value of a neutral solution, from each P_H value in turn, making the sign +, and raising 10 to the power indicated; the figures derived from P_H values less than 7 going in the acid, and from P_H values greater than 7 in the alkaline column.

For the two preceding determinations, 2-gram samples of the soils, in their natural condition, were treated in 50 cc. centrifuge tubes with 10 cc. of distilled water which had been boiling for some time in a resistance-glass flask to remove carbon dioxide. The tubes were stoppered, allowed to stand for several hours, and then centrifuged. Five cc. of the liquid, which represents the soluble matter in 1 gram of soil, was titrated in a resistance-glass vessel, using phenolphthalein as an indicator, carbon dioxide being continually boiled out. The remainder of the liquid was used for the color comparison, phenolsulfonephthalein and rosolic acid being found to be the most convenient indicators. The solutions of standard concentration required were tested electrometrically by Dr. Gillespie, to whom the writer's hearty thanks are herewith extended for this courtesy, as well as for much valuable advice in connection with the methods of procedure.

Because of the fact that phenolphthalein changes color only in a somewhat alkaline solution, the true alkalinity is always greater than the alkalinity inferred from the normality values; indeed, when the solution is only weakly alkaline, as in soils Nos. 3, 7, 13, 14, and 19, the two methods give apparently conflicting results, in that the reaction is acid toward phenolphthalein yet alkaline when compared to a truly neutral solution. It seems probable that the true alkalinity (or acidity) is of more significance than the normality, with reference to the growth of plants, and the normality determinations are given merely to permit comparison of the results with those which have been obtained in other similar investigations.

The rocks on which the walking fern was observed to grow are listed in Table 1 in order of decreasing lime content. Only

the first 5 are of types to which the term calcareous is generally applied, lime being present as carbonate, so that the rocks effervesce when acid is applied. The remainder comprise igneous,

TABLE 1

RESULTS OF ANALYSES

NO.	LOCALITY[a]	ROCK		SOIL						
		Name	Per cent lime	Per cent total lime	Per cent sol. lime	Normality		P_H	Intensities	
						Alk.	Acid		Alk.	Acid
1	3.5 mi. n. w. of L..	Limestone	53.8	5.4	0.21	0.006		8.5	30	
2	3.0 mi. s. w. of A..	Dolomite-limestone	29.0	10.5	0.05	0.011		9.0	100	
3	12.2 mi. s. w. of A..	Limestone-conglomerate	8.5	3.6	0.08		0.004	7.5	3	
4	12.3 mi. s. w. of A..	Limestone-conglomerate	8.3	2.3	0.06	neut	ral	8.0	10	
5	2.0 mi. s. of L.....	Argillaceous limestone	7.5	2.5	0.08		0.009	6.5		3
6	6.0 mi. s. w. of H..	Mica schist	7.5	6.3	0.04		0.006	7.0	neu	tral
7	10.7 mi. s. of A.....	Gneissoid granite	7.5	5.1	0.14		0.003	7.5	3	
8	3.0 mi. s. e. of A...	Gneissoid granite	7.2	4.3	0.03		0.007	7.0	neu	tral
9	1.3 mi. s. e. of H...	Mica schist	5.9	2.5	0.04		0.018	5.5		30
10	10.0 mi. n. w. of W.	Granitic gneiss	5.0	4.8	0.09		0.010	6.5		3
11	6.1 mi. s. w. of H..	Mica schist	4.1	2.0	0.07		0.017	5.5		30
12	11.0 mi. n. w. of A.	Shale	2.9	1.5	0.04		0.013	6.0		10
13	1.1 mi. s. e. of H..	Granite	2.0	4.3	0.04		0.004	7.5	3	
14	1.8 mi. n. of H.....	Mica schist	1.4	1.2	0.02		0.002	7.5	3	
15	12.5 mi. n. w. of A.	Sandstone	0.8	3.7	0.07		0.011	6.5		3
16	2.5 mi. s. of L.....	Shale	0.4	4.0	0.11		0.009	6.5		3
17	3.0 mi. s. of A.....	Jasperoid quartzite	0.1	5.1	0.10		0.018	5.5		30
18	0.2 mi. s. e. of H..	Quartzite	tr.	3.2	0.06		0.010	6.5		3
19	Same as 6.	Hemlock trunk		5.1	0.04		0.002	7.5	3	
20	Same as 1.	Hemlock stump		10.0	0.30	0.004		8.5	30	
Average for walking fern soils.....		....		4.4	0.08		0.008	7.0	neu	tral
Average for several field soils.....		....		0.8	0.01		0.015	6.0		10

[a] In this column the capital letters stand for the following localities:
A. = Allentown, Pa., H. = Harpers Ferry, W. Va., L. = Lewisburg, Pa., and W. = Washington, D. C.

metamorphic, and sedimentary rocks. Even though in some cases fairly high in lime, these are not ordinarily classed as calcareous, their lime being united with silica and other constituents in relatively insoluble form; several, indeed, contain only extremely small amounts of lime. When the total lime contents of the soils are compared to those of the rocks, however, wide divergences are shown. The several soils vary in total lime from 1.2 to 10.5 per cent, containing in some cases less and in others more lime than the rocks upon which they occur; in other words, in so far as this one constituent is concerned, the composition of the soil bears no relation whatever to that of the underlying rock. Soil transportation on any considerable scale is virtually excluded; but it is evident that leaching out of lime by the rain and its concentration by accumulation and decay of vegetable matter are both effective processes. Which is dominant depends on circumstances, such as exposure to rain, opportunity for leaves to fall in, etc., but the net result is that in cases where the rock is high in lime the soil usually contains less lime than the rock, whereas with rocks low in lime the reverse is usually true; and the average lime content of the soil is slightly more than 4 per cent. If a calcareous soil is defined as one containing more lime than the average field soil (0.8 per cent or less), then the soils supporting the growth of walking fern are certainly highly calcareous.

The percentage of soluble lime bears no recognizable relation to that of total lime, but it is also on the average many times as great as the figure for ordinary field soils. The habitat of the walking fern is thus calcareous with reference to both total and soluble lime. It is not claimed that the plant grows in these calcareous soils merely because of the presence of abundant lime; more probably the lime acts indirectly by favoring the accumulation of humus or the growth of beneficial microorganisms; no doubt the physical condition of the soils, the amount of moisture, the degree of drainage, etc., have an important bearing on the question also; but as this phase of the subject lies entirely outside of the scope of the present investigation it will not be further discussed.

The results in the last five columns, and more particularly the last two, show that some of the soils are alkaline, some neutral, and some acid. The chief sources of the alkalinity are, no doubt, calcium and potassium salts of weak organic acids; of the acidity, these acids in a free state; but the amount of soil available is too limited to permit of isolation or identification of these substances. At any rate it is evident that the walking fern is not especially sensitive to variations in the reaction of the soil.

Summary.—It has been shown by chemical analysis that the rocks supporting the growth of walking fern (*Camptosorus rhizophyllus*) are by no means necessarily calcareous, but that the soils in which this fern grows are rather high in both total and soluble lime. Rocks high in lime suffer leaching during soil formation, and those low in this constituent gain it through decay of vegetable matter, the ultimate amount varying widely with the conditions, but averaging about 4 per cent. The above results indicate that the view often held, that the occurrence of calciphilous plants necessarily indicates the presence of lime in the underlying rock strata, is untenable, except in cases where circumstances preclude the accumulation and decay of vegetable matter, and the resulting accumulation of lime in the soil.

ABSTRACTS

Authors of scientific papers are requested to see that abstracts, preferably prepared and signed by themselves, are forwarded promptly to the editors. Each of the scientific bureaus in Washington has a representative authorized to forward such material to this journal and abstracts of official publications should be transmitted through the representative of the bureau in which they originate. The abstracts should conform in length and general style to those appearing in this issue.

METALLURGY.—*The failure of brass.—1. Microstructure and initial stress in wrought brasses of the type 60 per cent copper and 40 per cent zinc.* P. D. MERICA and R. W. WOODWARD. Bureau of Standards Technologic Paper No. 82. Pp. 72. 1916.

This paper gives an account of an investigation of the cause of failure of a number of articles, particularly bolts, of wrought brass of the type 60 : 40 (i.e. of such material as naval brass and manganese bronze) with particular reference to the microstructure of the material and the presence in it of initial stress. In the course of this investigation the physical properties, microstructure, and initial stress distribution have been studied in some 250 materials, some of which had been in service (in the Catskill Aqueduct construction, in the Filtration Plant of the City of Minneapolis, in the U. S. Navy Department, and in the Panama Canal construction) and some of which was new material, rods having been kindly furnished by several manufacturers. It was shown that the initial stresses in rods could be relieved by annealing for one or two hours at low temperatures, 300°C. to 400°C., at which the physical properties of the rods were not appreciably affected.

P. D. M.

METALLURGY.—*The failure of brass.—2. The effect of corrosion on the ductility and strength of brass.* PAUL D. MERICA. Bureau of Standards Technologic Paper No. 83. Pp. 7. 1916.

Investigations of a homogeneous alpha brass have shown that the electrolytic solution potential of this material is increased by the application of a tensile stress. This measured increase amounts to approximately 0.1 millivolt for 10,000 lbs. per sq. in. of stress.

On this fact can be based an explanation of the decrease of strength and of ductility of brasses when corroded while under stress. Over a roughened surface of a bar the tensile stress will vary in value,

being greatest at the bottom of furrows and depressions and least, almost zero indeed, at the tops of the ridges. The e.m.f. will therefore, other things being equal, be greater (i.e., more electropositive) at the bottoms of the furrows than elsewhere; corrosion will set in here most rapidly, forming a crack which will grow narrower and sharper, its rate of growth being greater the sharper it is. In time the cross section of such a bar is so reduced by these cracks that fracture occurs, the brass exhibiting only slight elongation (ductility) and failing at a stress value apparently less than the ultimate strength. This explanation is borne out by the examination of a number of brass failures which have occurred under such conditions. P. D. M.

METALLURGY.—*The failure of brass.*—3. *Initial stress produced by the "burning-in" of manganese bronze.* PAUL D. MERICA and C. P. KARR. Bureau of Standards Technologic Paper No. 84. Pp. 7. 1916.

In connection with the failure, by cracking, of a number of manganese bronze valve castings in the Catskill Aqueduct at or near areas repaired by "burning-in," an investigation has been made of the initial stress produced in a manganese bronze double bar casting by the burning-in of a constrained portion. The stresses measured were in each case about 8000 to 10,000 pounds per square inch (i.e., the true elastic limit of the material) and the material within the burned-in area was of course in tension. The microstructure of the portion adjacent to the burned-in metal was not altered; the burned-in metal was in all cases of finer grain than that of the casting.

The conclusion is reached that, although distortion of a burned-in casting may partially relieve the initial stresses set up by this operation, such castings will, in all probability, generally contain local stresses of dangerous magnitude, i.e., near the elastic limit of the material. Castings repaired in this manner should either be thoroughly preheated or subsequently annealed in order to eliminate these stresses.

P. D. M.

GEOLOGY.—*Some Paleozoic sections in Arizona, and their correlation.* F. L. RANSOME. U. S. Geological Survey Professional Paper No. 98-K. Pp. 133–166, with 8 plates and 4 figures. 1916.

A comparison is made of ten stratigraphic sections from Bisbee, near the Mexican border, to the Grand Canyon. Attention is called to the thickness of the Cambrian beds in the Globe-Ray region in

contrast with those of the Grand Canyon section and to the exposure in the Mazatzal Range and Sierra Ancha of a thick series of quartzites, shales, and conglomerates unconformably beneath the Cambrian.

The observations presented in this paper indicate that in Cambrian time a land barrier existed in the region now adjacent to Tonto Basin, between the depositional basin of central and southern Arizona and that now corresponding to the Arizona Plateau, or at least to that part of the plateau between Payson and the Grand Canyon. It follows that while the whole Apache group of the Globe-Ray region and its stratigraphic equivalents in eastern and southern Arizona were deposited at probably about the same time as the beds of the Tonto group, the beds of the two groups were probably never continuous within the Tonto region of Arizona. It can not be said that any particular sandstone or quartzite of the Apache group is identical with the Tapeats sandstone.

Although the evidence from fossils is lacking, it appears to be fairly well established that the entire Apache group is Cambrian, or possibly in part younger, and, so far as can be seen over a wide region, is conformably overlain by the Devonian, while in the northern parts of the Mazatzal Range and Sierra Ancha the Paleozoic beds overlie with conspicuous unconformity a series of shales, quartzites, and conglomerates, which is probably equivalent to the Grand Canyon series.

The marked difference in the Carboniferous sections of the two geographic provinces in north-central Arizona suggests that the natural barrier supposed to exist in Cambrian time may have persisted in some form, possibly as a submarine ridge, throughout the Paleozoic era.

F. L. R.

BOTANY.—*Fungi of New Mexico.* PAUL C. STANDLEY. Mycologia, **8**: 142–177. 1916.

But little attention has been given by collectors to the lower plants of New Mexico. The only previous list of fungi of the state, enumerating 46 species, was published by Prof. T. D. A. Cockerell in 1904. The present paper is based chiefly on material collected by the author in connection with his work upon the phanerogamic plants. Two hundred and ten species are listed, 113 of which are rusts. There is included a description of a new rust, *Aecidium cockerellii* Arthur, on *Chamaesaracha coronopus* (Dunal) Gray. P. C. S.

PROCEEDINGS OF THE ACADEMY AND AFFILIATED SOCIETIES

BOTANICAL SOCIETY OF WASHINGTON

The 113th regular meeting of the Society was held in the Assembly Hall of the Cosmos Club, Tuesday, May 2, 1916. Mr. FRANK N. MEYER, geographical explorer of the U. S. Department of Agriculture, was elected to membership. The program consisted of the following papers.

Dr. Edward L. Greene, an appreciation: H. H. BARTLETT. As it was impossible for Mr. Bartlett to be present at the meeting, the paper was read by title. It has since been published in Torreya.[1]

Winter rape and adulterants of this seed (with lantern): EDGAR BROWN. Five types of plants raised from seed imported into the United States under the name of rape were briefly described and illustrated. The Dutch and German sources of the winter rape seed normally used in this country for the production of forage having been shut off, seed was imported from other sources, including winter rape from England, France, and Japan, annuals of no forage value from Argentina, France, China, and Japan, and biennials of no forage value from France and Japan.

An economic Amaranthus of ancient America (with exhibition of specimens and lantern): W. E. SAFFORD. Among the tributes paid to Montezuma by the pueblos of Mexico was a certain grain of ivory whiteness and more minute than a mustard seed, called by the Aztecs *huauhtli*. Eighteen imperial granaries were filled with it each year, each having a capacity of about 9000 bushels. In some parts of Mexico, at times when maize was scarce this seed was used in its stead and along the Pacific coast it was an important food staple. Cabeza de Vaca noticed it in Sonora in 1536. Its most important use was in religious ceremonies, when a paste, called *tzoalli*, was made of it together with maguey syrup, and images of the god Uitzilipuztli were molded of it. After having been adorned with beautiful ornaments and carried in procession, the image was carried to the top of the pyramidal temple in the city of Mexico. Sacrifices were made to it, including human beings, and the next day it was broken up into fragments and served as communion to the people. For a long time the botanical identity of this seed was unknown. The late Edward Palmer, while making col-

[1] **16**: 151–175, with portrait. July, 1916.

lections in the states of Sinaloa and Jalisco, found an *Amaranthus* growing both in cultivation and spontaneously. Its ivory-white seeds, resembling fish-eggs, corresponded exactly with the *huauhtli* as described by early writers. Moreover its local name, "guauto," is only a variation of the Nahuatl *huauhtli*. Near Guadalajara Dr. Palmer found a paste made of this seed and sugar offered for sale in the form of strings of dumplings enveloped in corn husks, under the name of "suale," a corruption of the Nahuatl *tzoali*. He collected botanical specimens of the plant producing the seed, which proved to be an *Amaranthus*, evidently a white-seeded form of *A. paniculatus*. Although Dr. Palmer did not realize that he had rediscovered an important economic plant of the Aztecs, his botanical specimens together with his field notes, found by the writer in the U. S. National Herbarium, have served to establish the identity of the sacred *huauhtli*. The possibility of cultivating this *Amaranthus* in suitable situations in the southwestern United States was suggested by the writer. Very closely allied plants, also producing white seeds, are cultivated as grain crops in India, Thibet, South America, and Africa. Of the existence of this particular form in pre-Columbian America there can be no doubt. It remains to be determined whether or not the Asiatic and African plants were endemic in the countries where they are now cultivated, or were introduced there after the discovery of America. Mr. Safford's paper will appear in full in the Proceedings of the Ninth Congress of Americanists.

Fungus fairy rings in eastern Colorado and their effect on vegetation (with lantern): H. L. SHANTZ and R. L. PIEMEISEL. (To be published by the U. S. Department of Agriculture.)

Report on the local flora: A. S. HITCHCOCK.

W. E. SAFFORD, *Corresponding Secretary.*

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